10.6 Segment Relationships in Circles

**Essential Question** What relationships exist among the segments formed by two intersecting chords or among segments of two secants that intersect outside a circle?

**EXPLORATION 1** Segments Formed by Two Intersecting Chords

*Work with a partner.* Use dynamic geometry software.

a. Construct two chords $BC$ and $DE$ that intersect in the interior of a circle at a point $F$.

b. Find the segment lengths $BF$, $CF$, $DF$, and $EF$ and complete the table. What do you observe?

<table>
<thead>
<tr>
<th>$BF$</th>
<th>$CF$</th>
<th>$BF \cdot CF$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DF$</td>
<td>$EF$</td>
<td>$DF \cdot EF$</td>
</tr>
</tbody>
</table>

c. Repeat parts (a) and (b) several times. Write a conjecture about your results.

**EXPLORATION 2** Secants Intersecting Outside a Circle

*Work with a partner.* Use dynamic geometry software.

a. Construct two secants $BC$ and $BD$ that intersect at a point $B$ outside a circle, as shown.

b. Find the segment lengths $BE$, $BC$, $BF$, and $BD$, and complete the table. What do you observe?

<table>
<thead>
<tr>
<th>$BE$</th>
<th>$BC$</th>
<th>$BE \cdot BC$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$BF$</td>
<td>$BD$</td>
<td>$BF \cdot BD$</td>
</tr>
</tbody>
</table>

c. Repeat parts (a) and (b) several times. Write a conjecture about your results.

**Communicate Your Answer**

3. What relationships exist among the segments formed by two intersecting chords or among segments of two secants that intersect outside a circle?

4. Find the segment length $AF$ in the figure at the left.
What You Will Learn

- Use segments of chords, tangents, and secants.

**Using Segments of Chords, Tangents, and Secants**

When two chords intersect in the interior of a circle, each chord is divided into two segments that are called segments of the chord.

**Theorem**

**Segments of Chords Theorem**

If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

**Proof** Ex. 19, p. 618

**EXAMPLE 1** Using Segments of Chords

Find $ML$ and $JK$.

**SOLUTION**

$NK \cdot NJ = NL \cdot NM$

$x \cdot (x + 4) = (x + 1) \cdot (x + 2)$

$x^2 + 4x = x^2 + 3x + 2$

$4x = 3x + 2$

$x = 2$

Find $ML$ and $JK$ by substitution.

$ML = (x + 2) + (x + 1)$

$= 2 + 2 + 1$

$= 7$

$JK = x + (x + 4)$

$= 2 + 2 + 4$

$= 8$

So, $ML = 7$ and $JK = 8$.

**Monitoring Progress**

Find the value of $x$.

1. $x \quad 6$

2. $\frac{4}{x + 1}$

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Core Concept

Tangent Segment and Secant Segment

A **tangent segment** is a segment that is tangent to a circle at an endpoint.

A **secant segment** is a segment that contains a chord of a circle and has exactly one endpoint outside the circle. The part of a secant segment that is outside the circle is called an **external segment**.

Theorem

Segments of Secants Theorem

If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.

**Proof**: Ex. 20, p. 618

EXAMPLE 2 Using Segments of Secants

Find the value of \(x\).

\[
RP \cdot RQ = RS \cdot RT
\]

\[
9 \cdot (11 + 9) = 10 \cdot (x + 10)
\]

\[
180 = 10x + 100
\]

\[
80 = 10x
\]

\[
x = 8
\]

The value of \(x\) is 8.

Monitoring Progress

Find the value of \(x\).

3.

\[
9 \quad x \quad 6
\]

4.

\[
3 \quad x + 2 \quad x + 1 \quad x - 1
\]
Segments of Secants and Tangents Theorem
If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.

Proof: Exs. 21 and 22, p. 618

**EXAMPLE 3** Using Segments of Secants and Tangents

Find $RS$.

**SOLUTION**

\[ RQ^2 = RS \cdot RT \]

\[ 16^2 = x \cdot (x + 8) \]

\[ 256 = x^2 + 8x \]

\[ 0 = x^2 + 8x - 256 \]

\[ x = \frac{-8 \pm \sqrt{8^2 - 4(1)(-256)}}{2(1)} \]

\[ x = -4 \pm \frac{4\sqrt{17}}{2} \]

Use the positive solution because lengths cannot be negative.

\[ x = -4 + 4\sqrt{17} \approx 12.49, \text{ and } RS \approx 12.49. \]

**EXAMPLE 4** Finding the Radius of a Circle

Find the radius of the aquarium tank.

**SOLUTION**

\[ CB^2 = CE \cdot CD \]

\[ 20^2 = 8 \cdot (2r + 8) \]

\[ 400 = 16r + 64 \]

\[ 336 = 16r \]

\[ 21 = r \]

So, the radius of the tank is 21 feet.

**Monitoring Progress**

Find the value of $x$.

5. 6. 7.

8. WHAT IF? In Example 4, $CB = 35$ feet and $CE = 14$ feet. Find the radius of the tank.
1. **VOCABULARY** The part of the secant segment that is outside the circle is called a(n) _____________.

2. **WRITING** Explain the difference between a tangent segment and a secant segment.

### Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the value of \( x \). (See Example 1.)

3. \[
\begin{align*}
10 & \quad 12 \\
6 & \quad x
\end{align*}
\]

4. \[
\begin{align*}
10 & \quad x - 3 \\
18 & \quad 9
\end{align*}
\]

5. \[
\begin{align*}
x & \quad 8 \\
6 & \quad x + 8
\end{align*}
\]

6. \[
\begin{align*}
2x & \quad 15 \\
12 & \quad x + 3
\end{align*}
\]

In Exercises 7–10, find the value of \( x \). (See Example 2.)

7. \[
\begin{align*}
10 & \quad x \\
6 & \quad 8
\end{align*}
\]

8. \[
\begin{align*}
x & \quad 7 \\
x & \quad 4
\end{align*}
\]

9. \[
\begin{align*}
x - 2 & \quad 4 \\
x & \quad x + 4
\end{align*}
\]

10. \[
\begin{align*}
45 & \quad x \\
27 & \quad 50
\end{align*}
\]

In Exercises 11–14, find the value of \( x \). (See Example 3.)

11. \[
\begin{align*}
7 & \quad x \\
9 & \quad 12
\end{align*}
\]

12. \[
\begin{align*}
24 & \quad 12 \\
x & \quad x
\end{align*}
\]

13. \[
\begin{align*}
12 & \quad x \\
x + 4 & \quad x + 4
\end{align*}
\]

14. \[
\begin{align*}
\sqrt{3} & \quad 2 \\
x & \quad x
\end{align*}
\]

15. **ERROR ANALYSIS** Describe and correct the error in finding \( CD \).

\[CD \cdot DF = AB \cdot AF\]
\[CD \cdot 4 = 5 \cdot 3\]
\[CD \cdot 4 = 15\]
\[CD = 3.75\]

16. **MODELING WITH MATHEMATICS** The Cassini spacecraft is on a mission in orbit around Saturn until September 2017. Three of Saturn’s moons, Tethys, Calypso, and Telesto, have nearly circular orbits of radius 295,000 kilometers. The diagram shows the positions of the moons and the spacecraft on one of Cassini’s missions. Find the distance \( DB \) from Cassini to Tethys when \( AD \) is tangent to the circular orbit. (See Example 4.)
17. **MODELING WITH MATHEMATICS** The circular stone mound in Ireland called Newgrange has a diameter of 250 feet. A passage 62 feet long leads toward the center of the mound. Find the perpendicular distance $x$ from the end of the passage to either side of the mound.

![Image of Newgrange mound]

18. **MODELING WITH MATHEMATICS** You are designing an animated logo for your website. Sparkles leave point $C$ and move to the outer circle along the segments shown so that all of the sparkles reach the outer circle at the same time. Sparkles travel from point $C$ to point $D$ at 2 centimeters per second. How fast should sparkles move from point $C$ to point $N$? Explain.

![Image of logo design]

19. **PROVING A THEOREM** Write a two-column proof of the Segments of Chords Theorem.

**Plan for Proof** Use the diagram from page 614. Draw $AC$ and $DB$. Show that $\triangle EAC$ and $\triangle EDB$ are similar. Use the fact that corresponding side lengths in similar triangles are proportional.

20. **PROVING A THEOREM** Prove the Segments of Secants Theorem. (*Hint:* Draw a diagram and add auxiliary line segments to form similar triangles.)

21. **PROVING A THEOREM** Use the Tangent Line to Circle Theorem to prove the Segments of Secants and Tangents Theorem for the special case when the secant segment contains the center of the circle.

22. **PROVING A THEOREM** Prove the Segments of Secants and Tangents Theorem. (*Hint:* Draw a diagram and add auxiliary line segments to form similar triangles.)

23. **WRITING EQUATIONS** In the diagram of the water well, $AB$, $AD$, and $DE$ are known. Write an equation for $BC$ using these three measurements.

![Diagram of water well]

24. **HOW DO YOU SEE IT?** Which two theorems would you need to use to find $PQ$? Explain your reasoning.

![Diagram of circle with points $P$, $Q$, and $R$]

25. **CRITICAL THINKING** In the figure, $AB = 12$, $BC = 8$, $DE = 6$, $PD = 4$, and $A$ is a point of tangency. Find the radius of $\odot P$.

![Diagram with point $A$ as a point of tangency]

26. **THOUGHT PROVOKING** Circumscribe a triangle about a circle. Then, using the points of tangency, inscribe a triangle in the circle. Must it be true that the two triangles are similar? Explain your reasoning.

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**Maintaining Mathematical Proficiency**

Reviewing what you learned in previous grades and lessons

Solve the equation by completing the square. (*Section 4.4*)

27. $x^2 + 4x = 45$

28. $x^2 - 2x - 1 = 8$

29. $2x^2 + 12x + 20 = 34$

30. $-4x^2 + 8x + 44 = 16$